1. (a) State one disadvantage of using quota sampling compared with simple random sampling.

(1)

In a university 8% of students are members of the university dance club.

A random sample of 36 students is taken from the university.

The random variable X represents the number of these students who are members of the dance club.

(b) Using a suitable model for X, find

(i) $P(X = 4)$

(ii) $P(X \geqslant 7)$

(3)

Only 40% of the university dance club members can dance the tango.

(c) Find the probability that a student is a member of the university dance club and can dance the tango.

(1)

A random sample of 50 students is taken from the university.

(d) Find the probability that fewer than 3 of these students are members of the university dance club and can dance the tango.

(2)

a) Disadvantage of quota sampling compared with simple random sampling :

⇒ Not random 1

b) $X \sim B(36, 0.08)$ 1

(i) $P(X = 4) = 0.167387....$

$= 0.167$ (3 s.f.) 1

(ii) $P(X \geqslant 7) = 1 - P(X \leqslant 6)$

$= 1 - 0.977...$

$= 0.022233...$

$= 0.0222$ (3 s.f.) 1

(c) P(dance club $\cap$ dance tango) = 0.08×0.4

= 0.032 (1)

d) let T = dance club and dance tango

$T \sim B(50, 0.032)$ (1)

$P(T < 3) = P(T \leq 2)$

$= 0.785081...$

$= 0.785$ (3 s.f.) (1)

2. The discrete random variable X has the following probability distribution

x	a	b	c
$P(X = x)$	$\log_{36} a$	$\log_{36} b$	$\log_{36} c$

where

- a, b and c are distinct integers $(a < b < c)$
- all the probabilities are greater than zero

(a) Find

(i) the value of a

(ii) the value of b

(iii) the value of c

Show your working clearly.

(5)

The independent random variables X_1 and X_2 each have the same distribution as X

(b) Find $P(X_1 = X_2)$

(2)

a) Σ probabilities $= 1$: $\log_{36} a + \log_{36} b + \log_{36} c = 1$ (1)

$\log_{36} abc = 1$

$abc = 36^1$

$\therefore abc = 36$ (1)

All probabilities are greater than zero :

Hence, $a, b, c > 1$, since $\log_{36} 1 = 0$ (1)

$\Rightarrow$ if we take factor of 36, and a, b, c are distinct integers.

Smallest factor $= 2$. So, $a = 2$ (1)

Next smallest factor $= 3$. So, $b = 3$

$c = 36 \div (2 \times 3) = 6$ (1)

b)

x	a	b	c
$P(X_1)$	$\log_{36} a$	$\log_{36} b$	$\log_{36} c$
$P(X_2)$	$\log_{36} a$	$\log_{36} b$	$\log_{36} c$

$P(X_1 = X_2) = (\log_{36} a)^2 + (\log_{36} b)^2 + (\log_{36} c)^2$ 1

$= (\log_{36} 2)^2 + (\log_{36} 3)^2 + (\log_{36} 6)^2$

$= 0.0374137\ldots + 0.09398737\ldots + 0.25$

$= 0.38140\ldots$

$= 0.381 \text{ (3 s.f.)}$ 1

3 A company has 1825 employees.

The employees are classified as professional, skilled or elementary.

The following table shows

- the number of employees in each classification
- the two areas, A or B, where the employees live

	A	B
Professional	740	380
Skilled	275	90
Elementary	260	80

An employee is chosen at random.

Find the probability that this employee

(a) is skilled,
(1)

(b) lives in area B and is not a professional.
(1)

Some classifications of employees are more likely to work from home.

1 • 65% of professional employees in both area A and area B work from home
2 • 40% of skilled employees in both area A and area B work from home
3 • 5% of elementary employees in both area A and area B work from home

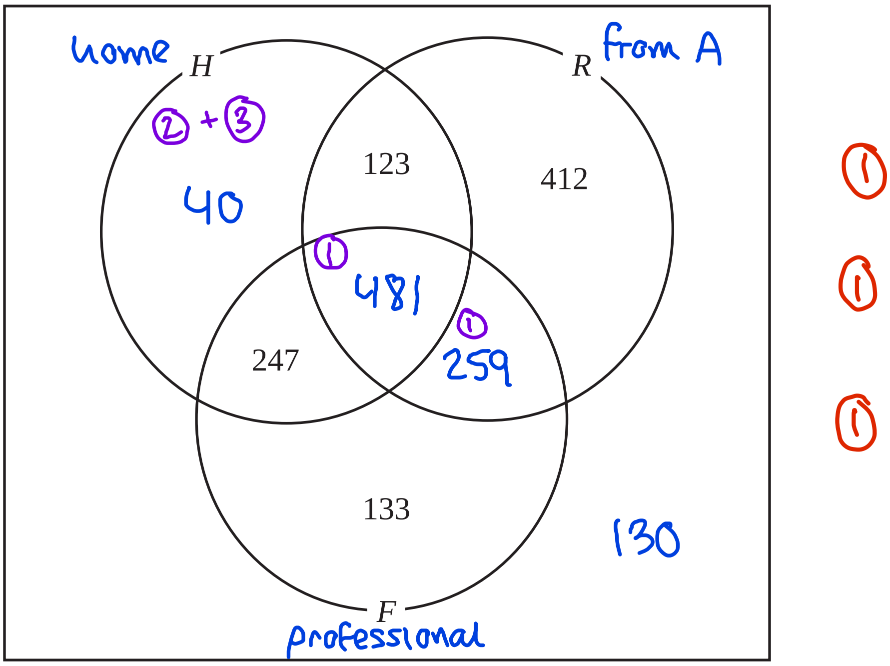
- Event F is that the employee is a professional
- Event H is that the employee works from home
- Event R is that the employee is from area A

(c) Using this information, complete the Venn diagram on the opposite page.
(4)

(d) Find $\mathrm{P}(R' \cap F)$
(1)

(e) Find $\mathrm{P}([H \cup R]')$
(1)

(f) Find $\mathrm{P}(F \mid H)$
(2)



Turn over for a spare diagram if you need to redraw your Venn diagram.

a) $P(\text{skilled}) = \dfrac{275+90}{1825} = \dfrac{1}{5}$ ①

b) $P(B \text{ and not professional}) = \dfrac{90+80}{1825} = \dfrac{34}{365}$ ①

c) ① 740 professional from A, 65% work from home
$740 \times 0.65 = 481$ ①

② 90 skilled from B, 40% work from home
$90 \times 0.4 = 36$

③ 80 elementary from B, 5% work from home
$80 \times 0.05 = 4$

$36 + 4 = 40$ non professionals from B who work from home

① 740 professional from A, 35% do not work from home: $740 \times 0.35 = 259$

$1825 - (40 + 123 + 412 + 481 + 247 + 259 + 133)$

$= 130$

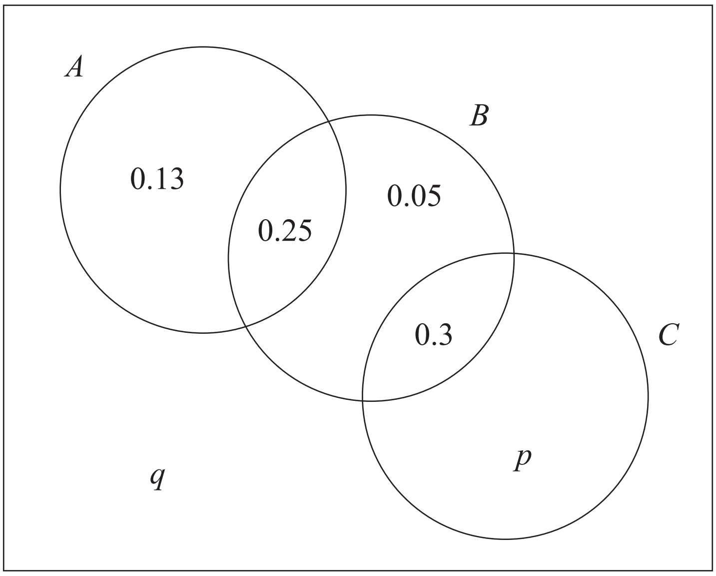
d) $P(R' \cap F) = \frac{247 + 133}{1825} = 0.208 \text{ (3sf)}$ (1)

e) $P([H \cup R]') = \frac{133 + 130}{1825} = 0.144 \text{ (3sf)}$ (1)

f) $P(F \mid H) = \frac{P(F \cap H)}{P(H)} = \frac{247 + 481}{40 + 123 + 247 + 481}$ (1)

$= 0.817 \text{ (3sf)}$ (1)

4. The Venn diagram, where p and q are probabilities, shows the three events A, B and C and their associated probabilities.



(a) Find P(A)

(1)

The events B and C are independent.

(b) Find the value of p and the value of q

(3)

(c) Find $P(A|B')$ $\rightarrow \dfrac{P(A \cap B')}{P(B')}$

(2)

a) $P(A) = 0.13 + 0.25$

$= 0.38$ (1)

b) $P(B \cap C) = P(B) \times P(C)$ — independent event

$0.3 = (0.3 + 0.25 + 0.05) \times (0.3 \times p)$ (1)

$0.3 = 0.6 \times (0.3 + p)$

$0.3 = 0.18 + 0.6p$

$0.12 = 0.6p$

$\therefore p = 0.2$ (1)

Σ probabilities $= 1$:

$0.13 + 0.25 + 0.05 + 0.3 + p + q = 1$

$0.13 + 0.25 + 0.05 + 0.3 + 0.2 + q = 1$

$0.93 + q = 1$

$\therefore q = 0.07$ (1)

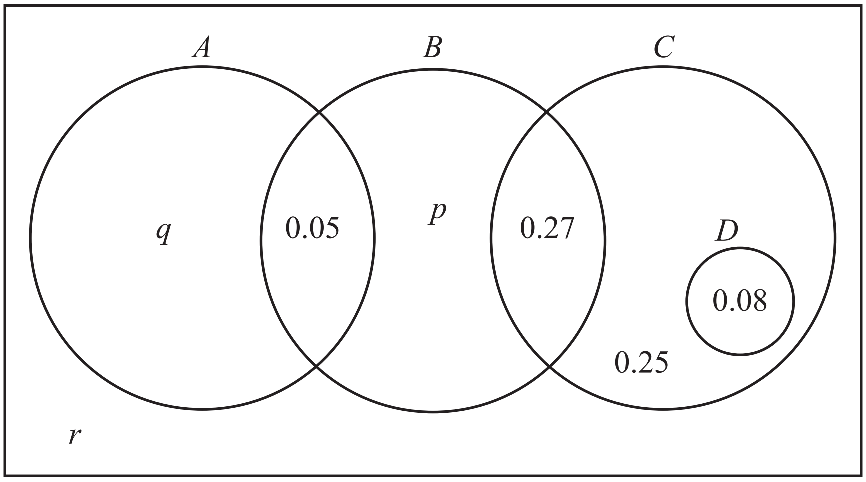
(c) $P(A \mid B') = \dfrac{P(A \cap B')}{P(B')}$

$= \dfrac{0.13}{0.13 + 0.2 + 0.07}$ (1)

$= \dfrac{0.13}{0.4}$

$= 0.325$ (1)

5. The Venn diagram, where p, q and r are probabilities, shows the events A, B, C and D and associated probabilities.



(a) State any pair of mutually exclusive events from A, B, C and D **(1)**

The events B and C are independent.

(b) Find the value of p **(2)**

(c) Find the greatest possible value of $\mathrm{P}(A \mid B')$ **(3)**

Given that $\mathrm{P}(B \mid A') = 0.5$

(d) find the value of q and the value of r **(3)**

(e) Find $\mathrm{P}\left([A \cup B]' \cap C\right)$ **(1)**

(f) Use set notation to write an expression for the event with probability p **(1)**

a) A and D ①

b) $P(B)P(C) = P(B \cap C)$

$P(C) = 0.6$ $\quad P(B) = p + 0.32$ $\quad P(B \cap C) = 0.27$ ①

$\Rightarrow 0.6(p + 0.32) = 0.27$

$p + 0.32 = 0.45$

$p = 0.13$ ①

c) $P(A|B') = \dfrac{P(A \cap B')}{P(B')}$

$= \dfrac{q}{q+r+0.25+0.08}$ ① $= \dfrac{q}{0.55}$

$q+r = 1-(0.05+0.13+0.27+0.25+0.08)$
$= 0.22$ ①

max $P(A|B') \longrightarrow$ max q

$r \geq 0$ so max q happens when $r=0 \Rightarrow q = 0.22$

max $P(A|B') = \dfrac{0.22}{0.55} = \dfrac{2}{5}$ ①

d) $P(B|A') = \dfrac{P(B \cap A')}{P(A')} = \dfrac{0.27+0.13}{0.6+0.13+r}$

$\Rightarrow \dfrac{0.4}{r+0.73} = 0.5$ ①

$r + 0.73 = 0.8$

$r = 0.07$ ①
$q = 0.22 - r$
$= 0.15$ ①

e) $P([A \cup B]' \cap C) = 0.25 + 0.08 = 0.33$ ①

"is not in A or B AND is in C"

f) $B \cap [A \cup C]'$ 1

"is not in A or C AND is in B"